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TI-59 PROGRAMS FOR MULTIPLE REGRESSION.(U)

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TI-59 PROGRAMS FOR

MULTIPLE REGRESSION

by

D. R. Barr

May 1980

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Programs for conducting multiple regression analyses with up to seven independent variables are given for the TI-59 calculator.

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TI-59 PROGRAMS FOR MULTIPLE REGRESSION

by

D. R. Barr

1. Introduction

The general linear hypothesis model of full rank [Graybill, 1961] can be written as

$$\underset{n\times 1}{Y} = \underset{n\times k}{x}\ \underset{k\times 1}{\beta} + \underset{n\times 1}{\varepsilon}\ , \qquad \varepsilon \sim N(0,\sigma^2 I)\ ,$$

where Y is the vector of n dependent variable values to be observed, X is the "design matrix" of fixed values of the independent variables, β is a vector of unknown fixed coefficients in the model and ε is the unobservable random error vector. The i^{th} row in this matrix equation,

$$Y_i = x_{0i}\beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + \varepsilon_i\ ,$$

represents the i^{th} dependent value to be observed as a linear function of the i^{th} set of independent variable values: $x_{0i}, x_{1i}, \cdots, x_{ki}$.

The solution to the normal equations for estimating β,

$$(x'x)\hat{\beta} = x'y\ ,$$

where y is the $n \times 1$ vector of observed outcomes on Y, can be found using program 02 in the TI-59 master module, provided $x'x$ is of full rank k and $1 < k < 9$. Due to space requirements for computing $x'x$, the program presented here requires $k < 8$. Also, because of optional programs (which run faster) for $k = 2$ (OP12-OP15 and $\Sigma+$) and $k = 3$ (statistics module program 18), the programs presented here are most useful for $3 < k < 8$. There is no limit on the number of observations, n. Tests of hypotheses about components of β can be carried out using a "reduced model" solution, and confidence intervals for linear functions of the coefficients can be obtained using $(x'x)^{-1}$ and $\hat{\sigma}^2$, based on the t-distribution, as outlined in [1]. Tests and confidence intervals for σ^2 can be obtained from $\hat{\sigma}^2$, based on the χ^2 distribution.

2. <u>The Programs</u>

Listings of two programs are given in the appendix. Program #1 serves two purposes:

(a) computation of $x'x$, $x'y$ and $y'y$ as rows of the data matrix $(x \vdots y)$ are entered;

(b) computation of $\hat{\beta}$, $\hat{\beta}'x'y$ and $(x'x)^{-1}$ for the full model.

Program #2 also serves two purposes:

(a) computation of reduced model components $(x'x)_r$ and $(x'y)_r$, and loading them into memory:

(b) computation of $\hat{\beta}_r'(x'y)_r$ for the reduced model.

A multiple regression problem is solved with these programs following the sequence:

(a) load Program #1 (2 banks)

(b) enter rows of the data matrix

(c) store $x'x$ and $x'y$ on cards

(d) compute and record $\hat{\beta}$, $\hat{\beta}'x'y$, $(x'x)^{-1}$, $y'y$ and $x'y$

(e) load Program #2 (1 bank), along with the stored data

(f) compute $\hat{\beta}_r'(x'y)_r$ for the reduced model.

Steps (c), (e) and (f) are optional, depending on whether it is desired to test hypotheses that various components of β are zero. Components of $\hat{\beta}$ and $(x'x)^{-1}$ can be recalled from memory (or listed on the printer using the "INV LIST" command) as desired.

Entering each row of the data matrix and updating $x'x$, $x'y$ and $y'y$ requires from 30-60 seconds, depending on k. Computation of $\hat{\beta}$ and $(x'x)^{-1}$ requires from 1 minute to 12 minutes, again depending on the dimension of $x'x$. In what follows, we give instructions for using the programs in the TI program record format. This is followed by an annotated example printer output (use of the printer is optional). Program listings are given in the Appendix.

3. Instructions

Instructions for running the programs are given in Figures 1 and 2. Using the quantities computed when both programs are used, an "analysis of variance" summary can be constructed. The format of this table, and sources of its entries, are given below, in Figure 3. An hypothesis of the form $H_0: \beta_i = 0$ is tested with the F ratio with the "β_i sum of squares" in the numerator and the "error mean square"

$$\hat{\sigma}^2 = \frac{y'y - \hat{\beta}'x'y}{n - k}$$

in the denominator. Degrees of freedom for this F ratio would be 1 and $n-k$. Note that the sum of squares due to regression in the full model, $\hat{\beta}'x'y$, generally is not partitioned into independent components due to $\beta_0, \beta_1, \ldots, \beta_k$ unless the design is "orthogonal" ($x'x$ is diagonal). Thus, tests of several hypotheses about components of β are generally not statistically independent. See [Searle, 1972] for a discussion of how to proceed if it is desired to test several hypotheses in the non-orthogonal case.

If one wishes to compute the coefficient of determination R^2 (which is not as useful as the related F-ratio), proceed as follows. First, compute the "adjusted total sum of squares," $SST_{(adj)} = y'y - (\Sigma y_i)^2/n$, where $y'y$ is stored in register 78, Σy_i in register 71* and n in register 08 at the first step of Program 1. Then

---

*See Comment in Figure 3.

TITLE MULTIPLE REGRESSION--PGM #1 PAGE 1 OF 1

PROGRAMMER BARR DATE 4/1/80

TI Programmable Program Record

Partitioning (Op 17) S.T.D. Library Module MASTER Printer OPTIONAL Cards 1 (Banks 1,2)

PROGRAM DESCRIPTION

For the general linear hypothesis model $Y = X\beta + \epsilon$, calculates: $x'x$, $x'y$, $y'y$, $|x'x|$, $(x'x)^{-1}$, $\hat{\beta}$, $\hat{\beta}'x'y$, Σy_i. Program can handle up to 7 independent variables. Running time increases as the dimension k of $x'x$ increases. Any number n of data points can be accommodated.

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Initialize	k	E	1
2	Enter rows of $(x \vdots y)$	x_{11} → R/S; x_{12} → R/S; ...; x_{1n} → R/S; y_1 → R/S	B	next row index
	(repeat above sequence until all rows entered).*			
3	Display $x'x$, $x'y$, $y'y$, Σy_i (or use "INV LIST" with printer)			
	#08 - $(07 + k^2)$: columns of $x'x$ (note: #08 is n)			
	#71 - $(70 + k)$: $x'y$. (NOTE: #71 is Σy_i.)			
	#78: $y'y$			
	Note: save $x'x$ and $x'y$ on cards if "reduced model" is desired (banks 2,3,4)			
4	Compute $\hat{\beta}$		C	1
	(with printer, $\|x'x\|$ is printed)			
5	Display $\hat{\beta}$, $\hat{\beta}'x'y$, $(x'x)^{-1}$: Recall (or use "INV LIST" w/printer)			
	#08 - $(07 + k^2)$: Columns of $(x'x)^{-1}$ in permuted order--order of permutation stored in #$(08 + k^2)$ - $(07 + k^2 + k)$			
	(#$08 + k^2 + k$) - $(07 + k^2 + 2k)$: $\hat{\beta}$			
	#71 - $(70 + k)$: $x'y$			
	#79: $\hat{\beta}'x'y$			

USER DEFINED KEYS	DATA REGISTERS		LABELS (Op 08) (see listing)
A	0	0	
B $x'x$ update	1 ALL USED;	1 INTERNAL	
C $\hat{\beta}$ computation	2 PARTITIONING	2 TO 319.79	
D	3	3	
E Initialize	4	4	

*If an error in entry is made, press "SBR 015" and re-enter the data row. This must be done before pressing "B".

FLAGS 0 1 2 3 4 5 6 7 8 9

FIGURE 1

TITLE MULTIPLE REGRESSION--PGM #2 PAGE 1 OF 1

TI Programmable
Program Record

PROGRAMMER BARR DATE 4/1/80

Partitioning (Op 17) 3,1,9,7,9 Library Module MASTER Printer OPTIONAL Cards 1 (Bank 1)

PROGRAM DESCRIPTION

For use with multiple regression program #1 to compute the reduced model components $(x'x)_r$ and $(x'y)_r$, and to compute the sum of squares due to regression in the reduced model, $\hat{\beta}_r'(x'y)_r$.

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter full data (x'x, x'y) into Banks 2, 3, 4.			
2	Load PGM #2 into Bank 1.			
3	Enter variable to be omitted		A	0
	(Repeat if several variables to be omitted-- omit in decreasing order of subscripts)			
4	Compute $\hat{\beta}_r'(x'y)_r$		B	$\hat{\beta}_r'(x'y)_r$

USER DEFINED KEYS	
A	Omit variables
B	Compute $\hat{\beta}_r'(x'y)_r$
C	
D	
E	
A'	
B'	
C'	
D'	
E'	

DATA REGISTERS (INV List): 0 00-79 used

LABELS (Op 08) (see listing)

FLAGS 0 1 2 3 4 5 6 7 8 9

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FIGURE 2

Index	Source	Degrees of Freedom	Sums of Squares
1.	Regression (full)	k	$\hat{\beta}'x'y$ (Pgm 1)
2_0.	β_0	1	$\hat{\beta}'x'y - \hat{\beta}_r'(x'y)_r$ (Pgms 1 & 2)
2_1.	β_1	1	
⋮	⋮		⋮
3.	Mean	1	$n\bar{y}^2$ (from Σy_i in first component of $x'y$, Pgm 1)*
4.	Error	n-k	$y'y - \beta'x'y$ (Pgm 1)
5.	Total	n	$y'y$

FIGURE 3. AOV Summary

*Provided the first column of X is a vector of 1's.

$$R^2 = \frac{SST_{(adj)} - SSE}{SST_{(adj)}} ,$$

where the sum of squares due to error is given by

$$SSE = y'y - \hat{\beta}'x'y = (n-k)\hat{\sigma}^2 .$$

4. Example. (k = 3 example from the TI "Applied Statistics" module booklet; page 5-15)

(1) Read in Program #1, banks 1 and 2,

(2) enter "3', press E,

(3) enter first row of $(x \vdots y)$, as follows (do not forget the "1" corresponding to the constant term):

1/ R/S, 0, R/S, 0, R/S, 17.3, R/S, B (next row index displays),

(repeat this sequence, entering 9 rows of data as in Table 5.2, which we produce in Table 1 for convenience.

row number	a_0	a_1	a_2	dependent variable
1	1	0	0	17.3
2	1	0	1	18.1
3	1	0	2	18.7
4	1	1	0	18.6
5	1	1	1	19.1
6	1	1	2	19.5
7	1	2	0	19.6
8	1	2	1	19.9
9	1	2	2	20.3

Table 1

(4) write banks 2, 3, 4 to save $x'x$, $x'y$ if tests of hypotheses about coefficients are desired (see listings below). Save $y'y$ (register 78) and Σy_i (register 71),

(5) press C to compute $\hat{\beta}$, $\hat{\beta}'x'y$, $(x'x)^{-1}$,

(6) recall and note desired components, as follows (see listing below)

a) columns of $(x'x)^{-1}$ in registers 08 to 16, order of columns permuted; permutation stored in 17, 18, 19

b) $\hat{\beta}$ in registers 20, 21, 22

c) $x'y$ in registers 71, 72, 73

d) $y'y$ (total sum of squares) in register 78

e) $\beta'x'y$ (sum of squares due to regression) in register 79.

(7) To test $H_0:a_0 = 0$ (for example) reduce model to eliminate the first independent variable as follows:

a) read in data (stored at step 2) in banks 2, 3, 4

b) read in Program #2, bank 1

c) enter the variable to be deleted ("1" in this case), press A

d) when computation is complete, Press B to compute the reduced sum of squares due to regression

e) when computation is complete, the sum of squares due to regression in the reduced model $\hat{\beta}_r'(x'y)_r$ (where "r" means "reduced"), is displayed and is stored in register 79.

f) The F-ratio for testing $H_0{:}a_0 = 0$ is formed as follows:

$$\frac{\hat{\beta}'x'y - \hat{\beta}_r'(x'y)_r}{(y'y - \hat{\beta}'x'y)/(n-k)} = 2705.2$$

degrees of freedom are 1 and $(n-k) = 6$.

(7') If it is desired to test other hypotheses of the form $H_0{:}a_i = 0$, repeat the steps in (7) above, with appropriate entry of "i" at step (7c). (Step (7b) need not be repeated.)

(7") It is desired to test a hypothesis that several coefficients are jointly zero, $H_0{:}a_i = a_j = \cdots = a_m = 0$, repeat step (7c) for each variable to be deleted, in descending order of magnitude of the subscripts. In this case, the F-ratio shown in (7f) should be divided by the number (say s) of remaining variables, and the F statistic has s and n-k degrees of freedom.

(8) Values obtained for the example and the hypothesis illustrated in step (7) are shown on the attached printouts. A summary AOV table is shown in Table 2.

Source	df	SS	F
Regression: full	3	3259.7161	
reduced	2	2565.8327	$\frac{693.88}{.1539/6} = 27$
due to a_0	1	693.8834	
Total	9	3259.87	
Error	6	(3259.87-3259.7161)=.1539	$\hat{\sigma}^2 = .1539/6$

Table 2. Summary AOV for example.

EXAMPLE RESULTS

At Step 4

	57.	00
	10.	01
	0.	02
	3.	03
	3.	04
	73.	05
	60.	06
k	3.	07
n	9.	08
	9.	09
	9.	10
	9.	11
x'x	15.	12
	9.	13
	9.	14
	9.	15
	15.	16
	0.	17
	0.	18
	0.	19

	0.	70
Σy_i	171.1	71
x'y	176.8	72
	174.1	73
	0.	74
	0.	75
	0.	76
	0.	77
yy'	3259.87	78
	0.	79

Step 5

|x'x| 324.

	9.	00
	16.	01
	16.	02
	19.	03
	3.	04
	4.	05
	324.	06
	3.	07
$x'x^{-1}$	-.1666666667	08
	-7. -13	09
	.1666666667	10
	-.1666666667	11
	.1666666667	12
	0.	13
	.4444444444	14
	-.1666666667	15
	-.1666666667	16
Permutation	3.	17
	2.	18
	1.	19
$\hat{\beta}$	17.56111111	20
	0.95	21
	0.5	22
	0.	23
	0.	24
	0.	25
	0.	70
	171.1	71
	176.8	72
	174.1	73
	0.	74
	0.	75
	0.	76
	0.	77
y'y	3259.87	78
$\hat{\beta}'x'y$	3259.716111	79

Step 7c-7e:

(enter "1", press "A")

	9.	00
	16.	01
	0.	02
	0.	03
	73.	04
	74.	05
	1.	06
	2.	07
reduced $x'x$ (omit a_0)	15.	08
	9.	09
	9.	10
	15.	11
	9.	12
	9.	13
	15.	14
	0.	15
	15.	16
	0.	17
	0.	18
	0.	19
	0.	20
	0.	70
reduced $x'y$	176.8	71
	174.1	72
	174.1	73
	0.	74
	0.	75
	0.	76
	0.	77
$y'y$	3259.87	78
	0.	79

(Press "B")

$|x'x_r|$ 144.

$\hat{\beta}_r'(x'y)_r$ 2565.832708

(displayed and stored in 79)

APPENDIX 1

PROGRAM LISTINGS

Note: Program #1 is recorded with standard partitioning; when run it internally partitions the calculator to 319.79. Program #2 and data cards storing x'x and x'y (if written) are written with 319.79 partitioning.

If it is desired to write Program #1 after it has been run, repartition back to standard format prior to writing (enter 6, press "2nd OP 17").

PROGRAM 1

LABEL ADDRESSES

001	15	E
009	18	C'
025	11	A
034	12	B
044	17	B'
104	16	A'
134	13	C
149	33	X²
157	34	ΓX
178	35	1/X
217	23	LNX
226	24	CE

MULTIPLE REGRESSION
PROGRAM 1

000	76	LBL
001	15	E
002	47	CMS
003	42	STD
004	07	07
005	08	8
006	69	OP
007	17	17
008	76	LBL
009	18	C'
010	73	RC*
011	06	06
012	33	X²
013	44	SUM
014	78	78
015	05	5
016	07	7
017	42	STD
018	00	00
019	69	OP
020	21	21
021	43	RCL
022	01	01
023	91	R/S
024	76	LBL
025	11	A
026	72	ST*
027	00	00
028	69	OP
029	20	20
030	91	R/S
031	61	GTO
032	11	A
033	76	LBL
034	12	B
035	22	INV
036	86	STF
037	01	01
038	00	0
039	42	STD
040	03	03
041	42	STD
042	04	04
043	76	LBL
044	17	B'
045	05	5
046	07	7
047	85	+
048	43	RCL
049	03	03
050	54	)
051	42	STD
052	05	05
053	05	5
054	07	7
055	85	+
056	43	RCL
057	04	04
058	54	)
059	42	STD
060	06	06
061	73	RC*
062	05	05
063	65	×
064	73	RC*
065	06	06
066	54	)
067	32	X:T
068	87	IFF
069	01	01
070	16	A'
071	08	8
072	85	+
073	43	RCL
074	04	04
075	85	+
076	53	(
077	43	RCL

```
078  07   07
079  65   ×
080  43  RCL
081  03   03
082  54   )
083  54   )
084  42  STO
085  05   05
086  32  X:T
087  74  SM*
088  05   05
089  69  OP
090  24   24
091  43  RCL
092  07   07
093  32  X:T
094  43  RCL
095  04   04
096  22  INV
097  77   GE
098  17  B'
099  86  STF
100  01   01
101  61  GTO
102  17  B'
103  76  LBL
104  16  A'
105  07   7
106  01   1
107  85   +
108  43  RCL
109  03   03
110  54   )
111  42  STO
112  05   05
113  32  X:T
114  74  SM*
115  05   05
116  69  OP
117  23   23
118  43  RCL
119  07   07
120  32  X:T
121  43  RCL
122  03   03
123  67   EQ
124  18  C'
125  00   0
126  42  STO
127  04   04
128  22  INV
129  86  STF
130  01   01
131  61  GTO
132  17  B'
133  76  LBL
134  13   C
135  43  RCL
136  01   01
137  75   -
138  01   1
139  54   )
140  42  STO
141  00   00
142  36  PGM
143  02   02
144  13   C
145  01   1
146  42  STO
147  03   03
148  76  LBL
149  33  X²
150  43  RCL
151  03   03
152  32  X:T
153  00   0
154  42  STO
155  04   04
156  76  LBL
157  34  ΓX
158  43  RCL
159  07   07
160  33  X²
161  85   +
162  08   8
163  85   +
164  43  RCL
165  04   04
166  54   )
167  42  STO
168  02   02
169  73  RC*
170  02   02
171  67   EQ
172  35  1/X
173  69  OP
174  24   24
175  61  GTO
176  34  ΓX
177  76  LBL
```

```
178  35  1/X
179  07  7
180  00  0
181  85  +
182  43  RCL
183  03  03
184  54  )
185  42  STO
186  02  02
187  08  8
188  85  +
189  43  RCL
190  07  07
191  85  +
192  43  RCL
193  07  07
194  33  X²
195  85  +
196  43  RCL
197  04  04
198  54  )
199  42  STO
200  05  05
201  73  RC*
202  02  02
203  72  ST*
204  05  05
205  43  RCL
206  07  07
207  32  X:T
208  43  RCL
209  03  03
210  67  EQ
211  23  LNX
212  69  OP
213  23  23
214  61  GTO
215  33  X²
216  76  LBL
217  23  LNX
218  25  CLR
219  36  PGM
220  02  02
221  15  E
222  00  0
223  42  STO
224  01  01
225  76  LBL
226  24  CE
227  08  8
228  85  +
229  43  RCL
230  07  07
231  33  X²
232  85  +
233  43  RCL
234  07  07
235  85  +
236  43  RCL
237  01  01
238  54  )
239  42  STO
240  02  02
241  07  7
242  01  1
243  85  +
244  43  RCL
245  01  01
246  54  )
247  42  STO
248  05  05
249  73  RC*
250  02  02
251  65  ×
252  73  RC*
253  05  05
254  54  )
255  44  SUM
256  79  79
257  69  OP
258  21  21
259  43  RCL
260  07  07
261  32  X:T
262  43  RCL
263  01  01
264  22  INV
265  77  GE
266  24  CE
267  25  CLR
268  36  PGM
269  02  02
270  17  B'
271  91  R/S
272  00  0
273  00  0
274  00  0
275  00  0
```

PROGRAM 2

LABEL ADDRESSES

032	11	A
072	17	B'
114	12	B
147	18	C'
178	19	D'

MULTIPLE REGRESSION

PROGRAM 2

000	76	LBL
001	16	A'
002	73	RC*
003	05	05
004	72	ST*
005	04	04
006	69	OP
007	25	25
008	69	OP
009	24	24
010	97	DSZ
011	03	03
012	16	A'
013	92	RTN
014	76	LBL
015	10	E'
016	43	RCL
017	07	07
018	75	-
019	43	RCL
020	06	06
021	54	)
022	65	×
023	43	RCL
024	07	07
025	54	)
026	42	STO
027	03	03
028	71	SBR
029	16	A'
030	92	RTN
031	76	LBL
032	11	A
033	42	STO
034	06	06
035	65	×
036	43	RCL
037	07	07
038	54	)
039	85	+
040	08	8
041	54	)
042	42	STO
043	05	05
044	75	-
045	43	RCL
046	07	07
047	54	)
048	42	STO
049	04	04
050	71	SBR
051	10	E'
052	08	8
053	85	+
054	43	RCL
055	06	06
056	54	)
057	42	STO
058	05	05
059	75	-
060	01	1
061	54	)
062	42	STO
063	04	04
064	43	RCL
065	07	07
066	75	-
067	02	2
068	54	)
069	42	STO
070	02	02
071	76	LBL
072	17	B'
073	43	RCL
074	07	07
075	75	-
076	01	1
077	54	)
078	42	STO
079	03	03
080	71	SBR
081	16	A'
082	69	OP
083	25	25
084	69	OP
085	32	32
086	43	RCL
087	02	02
088	32	X:T
089	00	0

```
090  22  INV
091  77   GE
092  17  B'
093  71  SBR
094  10  E'
095  07   7
096  01   1
097  85   +
098  43  RCL
099  06   06
100  54   )
101  42  STO
102  05   05
103  75   -
104  01   1
105  54   )
106  42  STO
107  04   04
108  43  RCL
109  07   07
110  75   -
111  43  RCL
112  06   06
113  54   )
114  42  STO
115  03   03
116  71  SBR
117  16  A'
118  69  OP
119  37   37
120  91  R/S
121  76  LBL
122  12   B
123  00   0
124  42  STO
125  79   79
126  36  PGM
127  02   02
128  13   C
129  43  RCL
130  07   07
131  42  STO
132  00   00
133  43  RCL
134  03   03
135  85   +
136  43  RCL
137  07   07
138  54   )
139  42  STO
140  01   01
141  76  LBL
142  18  C'
143  73  RC*
144  03   03
145  85   +
146  07   7
147  00   0
148  54   )
149  42  STO
150  04   04
151  73  RC*
152  04   04
153  72  ST*
154  01   01
155  69  OP
156  33   33
157  69  OP
158  31   31
159  97  DSZ
160  00   00
161  18  C'
162  69  OP
163  21   21
164  25  CLR
165  36  PGM
166  02   02
167  15   E
168  07   7
169  01   1
170  42  STO
171  02   02
172  76  LBL
173  19  D'
174  73  RC*
175  01   01
176  65   ×
177  73  RC*
178  02   02
179  54   )
180  44  SUM
181  79   79
182  69  OP
183  21   21
184  69  OP
185  22   22
186  97  DSZ
187  07   07
188  19  D'
189  43  RCL
190  79   79
191  91  R/S
192  00   0
193  00   0
194  00   0
195  00   0
```

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